The Grady Game

https://bit.ly/grady_game

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Abstract

This study investigates the effectiveness of visual and analytical methods in determining the minima of quadratic functions, utilizing data derived from a custom-developed game. The research aims to explore the influence of participants' self-reported cognitive styles—analytical, visual, or mixed—on their success in employing these methods. Success is defined by accuracy and efficiency in identifying minima, with game-derived metrics such as number of attempts and duration of each attempt aiding in comprehensive analysis. The study spans diverse demographic groups including the general population, high school students, and college students from STEM and humanities disciplines. This research contributes to understanding how individual differences in cognitive processing influence problem-solving in mathematical contexts, proposing that educational approaches should consider these differences to enhance learning outcomes.

Prior work

Visual and analytical methods in mathematical cognition are pivotal in shaping educational practices and enhancing learners' performance. Huincahue et al. (2021) illuminated the distinction between visual and analytical thinking styles in mathematics, emphasizing individual preference over inherent ability. The study delineated "mathematical thinking styles" (MTS) as the means by which learners engage with mathematical concepts, either through visual imaginings or formal representations. This is significant given the predilection for formal and analytical mathematics in evaluation processes, which has been shown to correlate with improved grades (Huincahue et al., 2021). Parallel to these findings, Zazkis et al. (1996) challenged the traditional dichotomy between visual and analytical strategies. Their research on the dihedral group D4 found that students employ a mix of both methods, positing that visualization and analysis are interdependent components of mathematical problem-solving. The proposed Visualizer/Analyzer (VA) model posits this interdependence and stresses the mutual dependency of both approaches, recommending that instructional designs accommodate this blend (Zazkis et al., 1996). Moreover, Godino et al. (2013)[3] explored the synergistic interplay between visual and analytical languages within mathematical thinking. Their work suggested that a deep understanding of both elements is fundamental for problem-solving and that educational approaches should foster the development of these complementary skills irrespective of the learner's cognitive style. Previous research by Rebekah M. Lane at Florida A & M University indicates that a balance between concept image (visual representations) and concept definition (linguistic formulation) led to a better understanding of functions, a notion supported by prior studies such as those by Thompson (1994) and Sfard (1991). The study's conclusions affirm that visual learners can benefit from instructional strategies that incorporate visual representations of mathematical concepts, facilitating a more profound comprehension and engagement with algebraic material. Collectively, these studies present compelling evidence for the interwoven nature of visual and analytical methods in mathematics. They advocate for a pedagogical framework that embraces the fusion of these approaches to cater to diverse cognitive preferences and enhance overall mathematical literacy and efficacy. This research builds upon such foundations, aiming to probe the correlation between cognitive styles and performance within a game-based environment, potentially extending the pedagogical implications of MTS in educational settings.

Approach

Quadratic Functions

Quadratic functions, typically written as $f(x) = ax^2 + bx + c$, are fundamental to various fields of mathematics, physics, and engineering. They describe parabolic curves and are essential for modeling motions, like the trajectory of objects under gravity. In economics, they can represent cost functions, while in physics, they're integral for detailing potential energy curves. The importance of locating their minimum (or maximum) is crucial, as it often represents optimal points such as the maximum height reached by a projectile or the most efficient level of production in a business scenario. Quadratic functions also arise in optimization problems and algorithms such as gradient descent, commonly used in the field of Artificial Intelligence.

Explanation of visual and analytical methods

Visual methods in solving quadratic functions involve graphing the parabola and identifying its lowest or highest point (the vertex) to find the minimum or maximum. This approach is intuitive and aids in understanding the function's behavior through its visual representation. Analytical methods, on the other hand, rely on mathematical techniques, such as completing the square or using the derivative (in calculus) to derive the vertex formula, allowing for the precise calculation of the function's minimum or maximum without graphing. Both approaches offer unique insights into the function's properties and optimal solutions.

Rationale

The rationale for this study is rooted in the observation that different cognitive styles may influence how individuals approach problem-solving in mathematical contexts. Given the ubiquity of quadratic functions across various disciplines and their significance in both academic and real-world scenarios, understanding whether a visual or analytical method is more effective in solving these functions–and how this effectiveness correlates with self-identified cognitive styles–could inform tailored educational strategies, enhance pedagogical tools, and deepen our comprehension of cognitive diversity in learning and problem-solving.

Experimental Setup

Research Questions

To determine the impact of visual vs. analytical approach for problem-solving, our study is guided by the following research question:

- Which method — visual or analytical — achieves a higher success rate in solving quadratic functions?

Additionally we want to take a look at the difference in performance among several demographic groups such as the general population, high school students, and college students from STEM or humanities disciplines?

Research Hypotheses

Based on our preliminary understanding of cognitive styles and their influence on problem-solving strategies, we propose the following null and alternative hypotheses:

H0: There is no statistical difference between success rates in visual mode games and analytical mode games, regardless of cognitive style. And our alternative hypothesis

H1: There is a significant difference in success rates in visual mode and success rates in analytical mode games.

We define success as a completed game with the minimal amount of guesses.

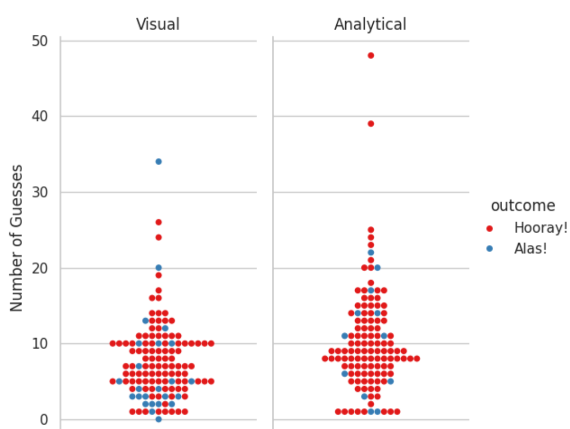
Data Collection

Data for the Grady game project is captured in real-time and stored in a structured log file, designed to facilitate comprehensive analysis of gameplay dynamics and outcomes. The data collection involves diverse cohorts including the general population, high school students, university students, social media users, and students from both the STEM and Humanities departments at Foothill College. This varied participant base enables the exploration of cognitive styles and problem-solving methods across different demographic and educational backgrounds. Each 'start' log entry records the initiation of a game and includes details such as the timestamp, player type, game type, and the coefficients of the quadratic function, along with its minimum value. 'Attempt' logs document each guess made by the player, capturing the guess number and the coordinates of the guess, along with the timestamp. Finally, 'final' logs summarize the game's outcome, noting the total number of guesses and the game result at completion.

Results and Discussion

For our statistical analysis we used data collected on player types, game types, number of guesses, and outcome of each game.

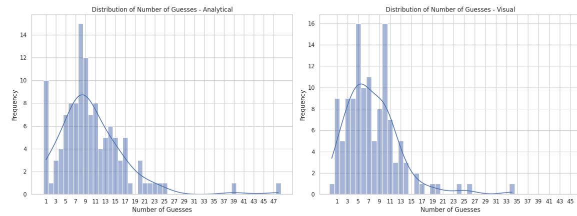
Outcomes and Number of Guesses



This graph demonstrates a few outliers in the Analytical mode guesses (38 and 47)

By taking a closer look at the frequency of guesses numbers, we can see that they are closely distributed.

Number of Guesses Distribution



Descriptive Statistics for Game Type

Game Type	Mean number of guesses	Std. Deviation
Visual	9.23	2.15
Analytical	9.15	2.20

Descriptive Statistics for Game Type

Self Reported Player Type	Mean number of guesses	Std. Deviation
Analytical	8.51	1.98
Visual	10.33	2.51
Both	9.44	2.12
Neither	9.35	2.25

For our global statistics the analysis revealed no significant interaction between cognitive styles and game modes on success, as indicated by the non-significant interaction term in the two-way ANOVA ($p = 0.42$). Additionally, the main effects of player type and game type on number of guesses were not significant ($p = 0.24$ and $p = 0.85$, respectively).

Two-Way ANOVA Results

Source	Sum Squares	Mean Squares	F-value	p-value
Player Type	34.52	11.51	2.61	0.24
Game Type	0.67	0.67	0.15	0.85
Interaction	10.89	2.17	0.49	0.42
Error	441.92	4.41	-	-

Post-hoc tests using Tukey's HSD revealed no significant differences in the number of guesses between player types (bird, bored, both, and nerd) or game types (analytical and visual). This suggests that the various player types and game types did not significantly influence the number of guesses made by participants, and the observed differences in means were likely due to chance.

Therefore, our data supports the null hypothesis that there is no difference in performance between the two game modes. This finding challenges the popular notion that individuals are either predominantly analytical or visual thinkers. It means that people possess a combination of both analytical and visual thinking abilities. Our research indicates that the human mind is more versatile and adaptable than previously thought.

In examining the interplay between cognitive styles and game completion rates across various populations, the comprehensive data from multiple tables provides a nuanced insight into behavioral patterns. Computer Science students, predominantly aligned with an analytical cognitive style, showed not only a higher proficiency in analytical games but also a tendency towards fewer guesses and quicker completion times. Specifically, Table 5 reveals that Computer Science students had the lowest median time to solve games at 0.23 minutes and a median of 5 guesses as per Table 6, underscoring their efficient problem-solving skills. In contrast, University Students, despite engaging less frequently in both visual and analytical games (Table 1), required a greater number of guesses (average of 15.13) and more time (average of 2.03 minutes) to complete games, indicating a potential mismatch between their cognitive styles and the chosen methods.

Furthermore, High School Students displayed a broader engagement across both game types, with a higher number of unsuccessful outcomes ("Alas!") in analytical games compared to their successes in visual games, as shown in Table 4. This might suggest that while their engagement is high, the effectiveness of their problem-solving approach varies by the cognitive demands of the game. The general population and Social Media Users demonstrated a stronger performance in visual games, which coincides with a higher completion rate (Table 3) and a favorable outcome ("Hooray!") in the majority of these games (Table 4).

The discrepancy between the global statistics and the population-specific data can be explained by the fact that the global analysis aggregates data from all populations, potentially masking the nuances and variations within each group. While the global analysis suggests no significant difference between visual and analytical thinkers, the population-specific data reveals that certain groups may have preferences or advantages in one cognitive style over the other.

In conclusion, the global statistics analysis indicates that there is no significant difference in performance between visual and analytical thinkers when considering the entire dataset. However, the population-specific data provides a more granular view, highlighting the variations in engagement, performance, and success rates among different groups based on their cognitive styles and the types of games they play. This emphasizes the importance of considering individual differences and population-specific factors when analyzing problem-solving strategies and game performance.

Further work

Our study on the relationship between cognitive styles and game performance faced several limitations that should be addressed in future research. Firstly, the success of our data collection relied heavily on the cooperation of various stakeholders, such as teachers, professors, administrators, and club officers. Their occasional unresponsiveness made it challenging to share the game with specific cohorts within the intended timeframe, leading to temporal overlap between cohorts and complicating data analysis. To mitigate this issue, future studies should establish clear communication channels and protocols with stakeholders well in advance, ensuring their commitment to facilitating timely data collection.

Secondly, the lack of a standardized sharing procedure may have influenced participants' performance. The content of the share message was tailored to different audiences, and some participants received more information about the game than others, particularly those in direct contact with the researchers. Additionally, some participants reported initial confusion due to the minimal instructions provided, which likely affected their performance, especially in their first attempts. To address this limitation, future research should develop a consistent and comprehensive set of instructions and share messages to ensure that all participants receive the same level of information and guidance.

Thirdly, our group's reach was limited, particularly in Shanghai, where we had few personal networks apart from university-level exchange students. In the U.S., our networks were primarily confined to Bay Area high schools and UC institutions. Moreover, as the researchers are studying STEM fields, the population was skewed towards individuals in STEM, resulting in a disproportionate number of "Nerd" responses, despite the self-selected and non-rigorous nature of the response. Future studies should aim to expand their reach to a more diverse population, collaborating with researchers and institutions across various geographical locations and disciplines to ensure a more representative sample.

Lastly, to maximize our reach, we avoided collecting personally identifiable information (PII). While this approach facilitated broader participation, it prevented us from differentiating between games played by the same person and those played by different individuals, potentially skewing our results towards STEM individuals. Future research should consider implementing methods to track individual participants while maintaining their privacy, such as assigning unique anonymous identifiers. This would allow for a more accurate analysis of individual performance and help control for potential biases.

Furthermore, the high game abandonment rate (over 50%) may have significantly impacted our results. An analysis of the abandoned games revealed that, on average, players made fewer guesses before abandoning the game compared to those who completed it. To address this issue, future studies should consider implementing controlled trials with a lower abandonment rate, possibly by providing incentives for completion or designing the game to be more engaging. This would provide a more accurate representation of the population's performance and help to minimize potential biases introduced by self-selection.

In conclusion, while our study provided valuable insights into the relationship between cognitive styles and game performance, future research should address the limitations we encountered. By establishing clear communication protocols, standardizing sharing procedures, expanding reach to diverse populations, implementing methods to track individual participants, and designing controlled trials with lower abandonment rates, future studies can build upon our findings and provide a more comprehensive understanding of how cognitive styles influence problem-solving in game-based environments.

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Appendix A: Tables

Table 1: Number of Games Started by Game Type

Population / Game Type	General Population	University Students	High School Students	CS Students	Social Media Users	FH Humanities	FH STEM
Visual	30	5	19	5	20	12	24
Analytical	24	3	9	4	29	11	24

Table 2: Number of Games Started by Player Type

Population / Player Type	General Population	University Students	High School Students	CS Students	Social Media Users	FH Humanities	FH STEM
Nerd	26	5	15	6	8	12	13
Bird	8	1	6	0	13	2	12
Both	6	1	2	1	12	3	12
Bored	14	1	5	2	16	6	11

Table 3: Game Completion

Population / Completion	General Population	University Students	High School Students	CS Students	Social Media Users	FH Humanities	FH STEM
Finished	54	8	28	9	49	23	0
Abandoned	54	25	25	2	72	35	48

Population / Outcome	General Population	University Students	High School Students	CS Studen
Alas!	6	0	21	1
Hooray!	48	8	7	8

Table 4: Number of Games Finished by Outcome

Table 5: Time Statistics to Solve the Game

Population / Stats	General Population	University Students	High School Students	CS Students	Social Media Users	FH Humanities	FH STEM
Min	0.06	0.46	0.05	0.03	0.04	0.05	0.05
Max	5.20	4.80	3.84	2.58	6.95	4.83	17.42
Median	0.82	2.13	0.93	0.23	0.71	0.90	1.31
Average	1.28	2.02	1.34	0.58	1.04	1.33	1.97

Table 6: Number of Guesses Statistics

Population / Stats	General Population	University Students	High School Students	CS Students	Social Media Users	FH Humanities	FH STEM
Min	1	8	1	1	1	1	1
Max	24	39	34	20	25	20	48
Median	9	11.5	7	5	8	8	9
Average	9.94	15.13	7.93	6.78	8.33	7.52	10.31